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Analysis of Students' Errors Based on Newman's Errors in Solving Mathematical Reasoning Problems

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Abstract- Mathematical reasoning is one of the essential competencies that students must possess to face the challenges of 21st-century learning. However, many students still struggle to solve problems that assess their mathematical reasoning. This study aims to analyse the types of errors students make using Newman's Error Analysis procedure. The research employed a descriptive qualitative method, with 37 twelfth-grade high school students selected through purposive sampling. Data were collected through a mathematical reasoning ability test and interviews. The quantitative data from the test were analysed descriptively to identify error patterns, while the qualitative data from interviews were analysed thematically through data reduction, data display, and conclusion drawing. Most students made errors in three main stages, namely understanding the problem concept, transforming verbal information into appropriate mathematical representations, and applying mathematical procedures or algorithms correctly. Furthermore, the analysis showed that conceptual misunderstanding was the most prevalent type of error, often leading to subsequent transformation and process-skill errors. These findings suggest that teachers need to design learning interventions that strengthen conceptual comprehension and develop students' systematic problem-solving strategies.

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1. Introduction

Mathematical reasoning skills occupy a central position in various national and international curriculum documents as a higher-order thinking skill.

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that students must possess in the 21st century (Özaydin & Arslan, 2022; Putra et al., 2024). The Merdeka Curriculum emphasises strengthening literacy, numeracy, and reasoning as part of competency-based learning. Mathematical reasoning skills are seen as an important means of developing logical, critical, and systematic thinking patterns in problem-solving and evidence-based decision-making (Kemdikbudristek, 2022). The Merdeka Curriculum positions the Pancasila Student Profile as the primary orientation of education, encompassing elements of critical thinking and global diversity (Brilliananda et al., 2025). Mathematical reasoning serves as a vehicle for training students to process information, make inferences, and construct logically accountable arguments (Mumu & Tanujaya, 2019). In mathematics learning outcomes, reasoning ability is explicitly stated as one of the learning objectives at both the primary and secondary levels, through activities such as concluding, generalising, and proving (Kemdikbudristek, 2022).

NCTM (2000) mentions reasoning as one of the five important processes in mathematics learning, along with problem solving, communication, connection, and representation. Mathematical reasoning in this context includes the ability to understand and construct mathematical arguments, to test their validity, and to generalise patterns and structures. A similar emphasis is placed on reasoning ability by the Program for International Student Assessment (PISA), which uses it as a key indicator of students' mathematical literacy across countries (OECD, 2019). Students with strong reasoning skills can tackle real-world problems with an analytical and data-driven approach. PISA defines mathematical literacy as an individual's capacity to formulate, use, and interpret mathematics in various contexts, including the ability to reason and argue mathematically (OECD, 2019).

Although mathematical reasoning skills are the main focus of mathematics learning, the reality is that most high school students struggle to solve problems that test these skills (Akrom et al., 2021; Negara et al., 2024). Mathematical reasoning requires a series of higher-order thinking processes, such as understanding information, connecting concepts, constructing logical arguments, and drawing conclusions or proving statements. Unfortunately, not all students possess the cognitive skills necessary to carry out these processes systematically (Putra et al., 2024; Sumarmo, 2010). Students often fail to understand the meaning of mathematical problems, even before they can perform more complex reasoning. Students experience confusion when interpreting questions, connecting the information provided to relevant mathematical concepts, and designing logical steps to solve problems. This indicates that students' weaknesses lie in their mastery of concepts, ways of thinking, and reasoning.

Based on PISA 2018 data, Indonesian students' performance in mathematics is relatively low, particularly on questions requiring reasoning, modelling, and reflection on solution strategies. Most students complete questions at a low cognitive level (e.g., calculation or direct application of formulas) but struggle with questions that require interpretation, generalisation, or argumentation (OECD, 2019; Rum & Juandi, 2022). At the national level, the results of the National Assessment show that the majority of students fall into the category requiring special intervention for critical thinking and numeracy indicators. One cause is the lack of learning that stimulates the reasoning process and focuses on technical procedures or formula memorisation (Herbert, 2021). A mechanistic learning approach with minimal reflective dialogue further exacerbates students' low proficiency in solving mathematical reasoning problems. This situation underscores the need for an in-depth analysis of students' errors to identify the root causes of their difficulties (Dewantara et al., 2024; Valdez & Taganap, 2024).

One relevant approach to analysing student errors is Newman Error Analysis (NEA), developed by M. A. Newman in 1977. This procedure classifies student errors into Reading Error (RE), Comprehension Error (CE), Transformation Error (TE), Process Skill Error (PE), and Encoding Error (EE) (Newman, 1977). The NEA approach provides a systematic framework for identifying students' points of failure in solving mathematical problems (Kurniati et al., 2021). The novelty of this study lies in its application of the NEA framework specifically to analyse errors in mathematical reasoning tasks among twelfth-grade students. This context has received limited attention in previous studies. By focusing on the types and causes of reasoning errors, this study contributes to a deeper understanding of students' cognitive processes. It provides empirical insights for developing more targeted and responsive instructional strategies.

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Error analysis based on NEA is becoming increasingly important in contemporary mathematics education, which emphasises higher-order thinking processes (Makamure & Jojo, 2022; Valdez & Taganap, 2024). By identifying the types and frequencies of errors made by students at each stage of problem-solving, teachers can more accurately diagnose learning difficulties and design appropriate instructional interventions (Baki & Gürsoy, 2020; Özdemir & Dede, 2022; Pinzón et al., 2022). Moreover, understanding students' error patterns can foster their metacognitive awareness during the mathematics learning process (Adinda et al., 2021; Sercenia et al., 2023). Therefore, this study aims to analyse the types and causes of errors made by high school students when solving mathematical reasoning problems, using the NEA framework. Specifically, it seeks to identify which stages of Newman's procedure pose the greatest challenges for students and to provide pedagogical implications for improving instruction in mathematical reasoning.

2. Methods

This study employs a qualitative descriptive approach to describe and deeply understand the types of errors students make when solving mathematical reasoning problems, based on Newman Error Analysis (NEA). The qualitative approach was chosen because it aligns with the research objectives, which focus on exploring phenomena, interpreting meanings, and understanding students' cognitive and behavioural processes during mathematical problem solving rather than measuring variables statistically (Creswell & Creswell, 2018). This approach enables the researcher to capture students' errors in context and in detail, both in the problem-solving process and in the strategies employed.

The subjects of this study were 37 twelfth-grade high school students selected through purposive sampling. The selection criteria included students who had completed mathematical topics related to reasoning—such as quadratic functions, inequalities, and algebraic manipulation—and who represented varying levels of mathematical ability (high, medium, and low). The classification of ability levels was determined based on students' previous mathematics scores and teacher recommendations. In general, the participants were aged 16–18 years and had diverse academic backgrounds and learning motivations. This diversity allowed the researcher to identify a broad spectrum of error patterns and reasoning characteristics across different ability levels, providing a more comprehensive picture of students' mathematical reasoning difficulties (Miles et al., 2014).

The instruments used in this study consisted of a mathematical reasoning test and interview guidelines. The following is an excerpt from the questions given to students.

A city park is designed in the shape of a symmetrical parabola bounded by a curved fence. A quadratic function can model the curve of the fence. The length of the base of the park (x-axis) is 20 metres, and the highest point of the curved fence is exactly in the centre of the park at a height of 5 metres. The city government wants to know the following.

a. What is the quadratic function equation that models the shape of the curved fence?
b. What is the height of the fence when the distance from the left end of the park is 3 metres?
c. If the fence is set to have a minimum height of 1 metre, determine the length interval of the park that meets this requirement.
d. Explain why this quadratic function model is appropriate for the shape of the fence.
e. Is it possible to make the garden wider while maintaining a maximum height of 5 metres and a parabolic shape? Provide your guess and reasoning.

The students' answers were then categorised according to the categories in Table 1, followed by interviews with high-, medium-, and low-ability students.

Table 1. Student Answer Categories Based on NEA

Student's Answer	NEA
No answer.	RE
Incorrect in determining the quadratic function equation that models the curved shape of the garden fence.	CE
Incorrect determination of fence height and garden length intervals that meet certain requirements.	TE
Incorrect in performing algebraic calculations and explaining the reasons.	PE
Incorrect conclusion.	EE

The data analysis process was conducted through three stages: data reduction, data presentation, and conclusion drawing (Miles et al., 2014). In the data reduction stage, students' written responses were carefully examined and categorised according to Newman's five error types: Reading Error (RE), Comprehension Error (CE), Transformation Error (TE), Process Skill Error (PE), and Encoding Error (EE). Each student's response was analysed item by item to identify the specific stage at which the error occurred, with excerpts from interview transcripts used to clarify the reasoning behind the mistakes. During the data presentation stage, the results were organised into tables showing the frequency and percentage of each error type, accompanied by narrative explanations and representative examples of students' work for each category. Finally, in the conclusion-drawing stage, the researcher interpreted the findings by identifying the dominant error types, examining their underlying causes, and relating them to students' mathematical reasoning processes. To enhance the validity and reliability of the analysis, source triangulation was applied by comparing written test results with interview data, ensuring consistency between students' responses and their verbal explanations.

3. Results and Discussion

This error analysis was applied to students' work on mathematical reasoning questions. The questions were designed to explore seven indicators of reasoning ability, such as constructing mathematical statements, performing algebraic manipulations, constructing proofs, and drawing conclusions. This data provides an initial overview of the most common weaknesses students encounter in the mathematical thinking process. This information is crucial for evaluating learning outcomes and serves as a foundation for designing targeted, appropriate learning interventions tailored to students' difficulty profiles.

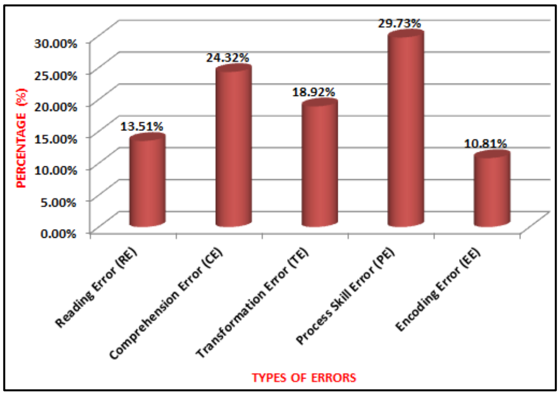

Figure 1. Distribution of Types of Student Errors

Based on Figure 1, PE is the most common error among students, accounting for 29.73% of respondents. This error indicates that students have difficulty performing algebraic manipulations, arithmetic operations, or other mathematical procedures required to solve quadratic function problems. This finding is consistent with Özaydin (Özaydin & Arslan, 2022), who states that symbolic manipulation is one of the main difficulties students face when solving mathematical reasoning problems. Weaknesses at this stage hinder systematic, logical thinking and become a crucial point of failure in problem-solving (Wekwete & Higgs, 2024).

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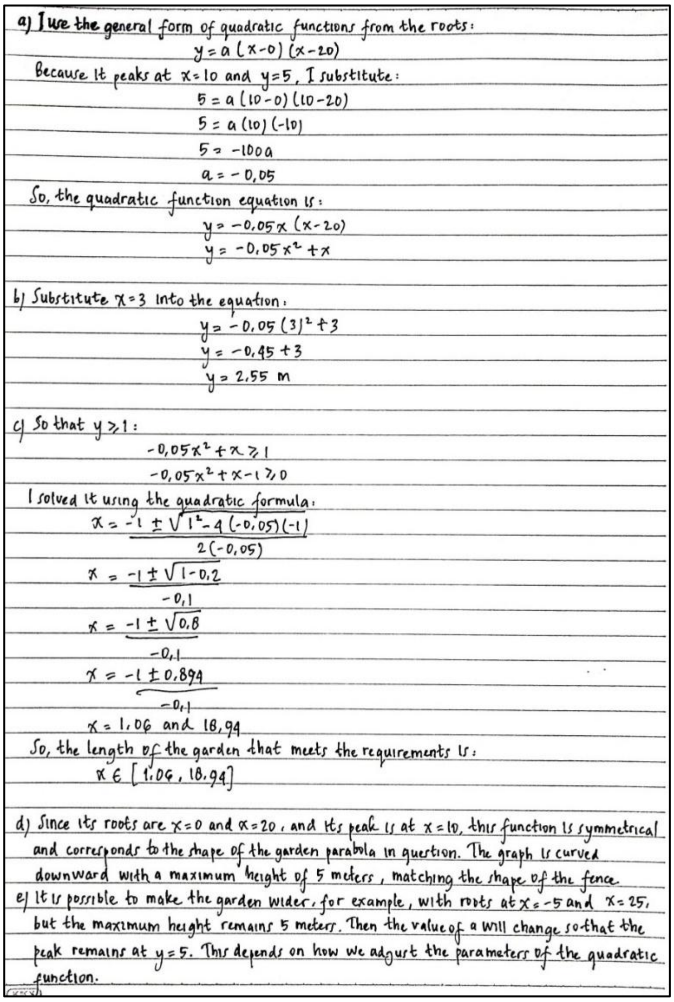
The second most common type of error was CE, which accounted for 24.32%. This error occurred when students failed to understand the meaning or information in the question, either explicitly or implicitly. Although students could read the question, they did not fully understand the problem's context or the information to be processed. This is consistent with Newman's (Newman, 1977) statement that CE can occur even when not preceded by RE. In contextual questions, understanding the real situation, presented as a story, is crucial because it forms the basis of the transformation and manipulation process. TE ranks third at 18.92%. This error refers to students' inability to transform the question statement into a mathematical model, such as constructing a quadratic equation from the available information. Ünal (2023) notes that the transformation from everyday language to symbolic representation is a major challenge in mathematics learning because it requires metacognitive skills and abstract reasoning.

RE and EE were the least common errors, accounting for 13.51% and 10.81%, respectively. RE reflects students' inability to read or recognise symbols and terms in questions due to low mathematical literacy. Meanwhile, EE occurs when students complete the solution process but fail to present the final answer correctly, due to errors in format, symbols, or reasoning (Valdez & Taganap, 2024; Zakaria et al., 2010). EE is caused by students' lack of habit in explicitly writing down their thinking process in mathematical form (Makgakga, 2023; Yarman et al., 2025). To explore further information, the researcher conducted interviews with students representing different abilities.

(a) High Ability Student (HAS)

Figure 2 shows HAS's systematic, logical thinking process, reflecting his mastery of mathematical concepts and reasoning skills. In the initial stage, HAS was able to understand the contextual information from the question describing a city park in the shape of a parabola. HAS understood the text literally and connected the geometric information (the intersection of the parabola with the x-axis) with the general form of a quadratic function. This indicates that the student has met the indicator of understanding and presenting mathematical statements in the form of images and symbols (Matabane & Machaba, 2023; Ridho et al., 2023).



a) I use the general form of quadratic functions from the roots:
$y = a(x-0)(x-20)$
Because it peaks at $x=10$ and $y=5$, I substitute:
$5 = a(10-0)(10-20)$
$5 = a(10)(-10)$
$5 = -100a$
$a = -0{,}05$
So, the quadratic function equation is:
$y = -0{,}05x(x-20)$
$y = -0{,}05x^2 + x$

b) Substitute $x=3$ into the equation:
$y = -0{,}05(3)^2 + 3$
$y = -0{,}45 + 3$
$y = 2{,}55$ m

c) So that $y \geq 1$:
$-0{,}05x^2 + x \geq 1$
$-0{,}05x^2 + x - 1 \geq 0$
I solved it using the quadratic formula:
$x = \frac{-1 \pm \sqrt{1^2 - 4(-0{,}05)(-1)}}{2(-0{,}05)}$
$x = \frac{-1 \pm \sqrt{1-0{,}2}}{-0{,}1}$
$x = \frac{-1 \pm \sqrt{0{,}8}}{-0{,}1}$
$x = \frac{-1 \pm 0{,}894}{-0{,}1}$
$x = 1{,}06$ and $18{,}94$
So, the length of the garden that meets the requirements is:
$x \in [1{,}06, 18{,}94]$

d) Since its roots are $x=0$ and $x=20$, and its peak is at $x=10$, this function is symmetrical and corresponds to the shape of the garden parabola in question. The graph is curved downward with a maximum height of 5 meters, matching the shape of the fence.
e) It is possible to make the garden wider, for example, with roots at $x=-5$ and $x=25$, but the maximum height remains 5 meters. Then the value of a will change so that the peak remains at $y=5$. This depends on how we adjust the parameters of the quadratic function.

Figure 2. HAS's Answer

Next, HAS made a reasonable assumption based on the available visual data and numerical information. HAS written down the general form of the quadratic function $y = a(x - x_1)(x - x_2)$ and entered the values of the intersection points (0, 0) and (20, 0) as roots. This process reflects the ability to transform verbal information into a mathematical model and demonstrates mastery of the basic principles of quadratic functions (Jupri, Usdiyana, et al., 2021; Ridho et al., 2023). When determining the value of the coefficient a, HAS used the points $x = 10$ and $y = 5$, accurately substituting them into the function model to find the value of a. This demonstrates the ability to perform mathematical manipulation and advanced algebraic skills (Jones & Tiller, 2017; Jupri, Sispiyati, et al., 2021). In the NEA, this student made no TE or PE errors, indicating a strong level of procedural mastery.

Furthermore, HAS included the reasons and proofs for his steps by explaining why the vertex of the parabola is located at $x = 10$. HAS verified his results by substituting several values into the function and checking whether the results were consistent. This step demonstrates the fulfilment of the indicator of constructing evidence and checking the validity of mathematical arguments (NCTM, 2000). HAS draws the mathematical conclusion that the parabola is symmetrical and that the maximum value is obtained on the axis of symmetry, a generalisation of the properties of quadratic functions.

Researcher	:	*When you see the question about the parabolic city park, what is the first thing that comes to mind?*
HAS	:	*I immediately saw the intersection points of the parabola with the x-axis at 0 and 20, sir. So, I thought the general form of the equation would use the root formula, namely* $y = a(x - x_1)(x - x_2)$.
Researcher	:	*How can you be sure that the intersection points 0 and 20 are the roots of the quadratic function?*

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HAS	:	*Because in the image, the ends of the garden fence are at x-coordinates 0 and 20. It means the y-values are zero at those points. So, I conclude that those are the roots, sir.*
Researcher	:	*Then how do you determine the value of a in that quadratic function?*
HAS	:	*I use the vertex of the parabola in the middle, which is the point (10, 5), sir. I substitute it into the formula* $y = a(x-0)(x-20)$, *so* $5 = a(10)(-10)$. *Then I calculate* $a = -0.05$.
Researcher	:	*You also wrote that this function is symmetrical. Can you explain why you think so?*
HAS	:	*Because the distance between the intersection points is the same from the center point, 0 to 10 and 10 to 20. So, I know that the axis of symmetry of the parabola is at* $x = 10$ *and this is also the maximum point. It is a general property of quadratic function graphs.*
Researcher	:	*Do you double-check your results?*
HAS	:	*Yes, sir, I tried entering the values* $x = 3$ *and* $x = 9$ *and the results were the same, symmetrical. The y-values were the same at both points, so I am sure my equation is correct, sir.*
Researcher	:	*In your opinion, which part of this question is the most challenging?*
HAS	:	*Determine the value of a, because you have to calculate carefully and make sure the sign is correct. But once you know the peak, the rest is just substitution.*

Overall, students in the high-ability group demonstrated a comprehensive mathematical reasoning profile that encompassed nearly all targeted indicators (Negara et al., 2024; Putra et al., 2024). They not only solved problems accurately but also reflected on the reasoning processes they employed, indicating a well-developed level of mathematical metacognition. This finding aligns with Polya's framework, which emphasises reflection as an essential component of effective problem-solving. HAS's ability to monitor and evaluate their reasoning process also supports the notion that metacognitive awareness plays a key role in achieving higher-order reasoning (NCTM, 2000; Polya, 1973). In the context of the Merdeka Curriculum, such reasoning behaviour represents the ideal learning outcome, in which students can connect concepts and apply them adaptively in various situations (Herbert, 2021; Mumu & Tanujaya, 2019).

(b) Medium Ability Student (MAS)

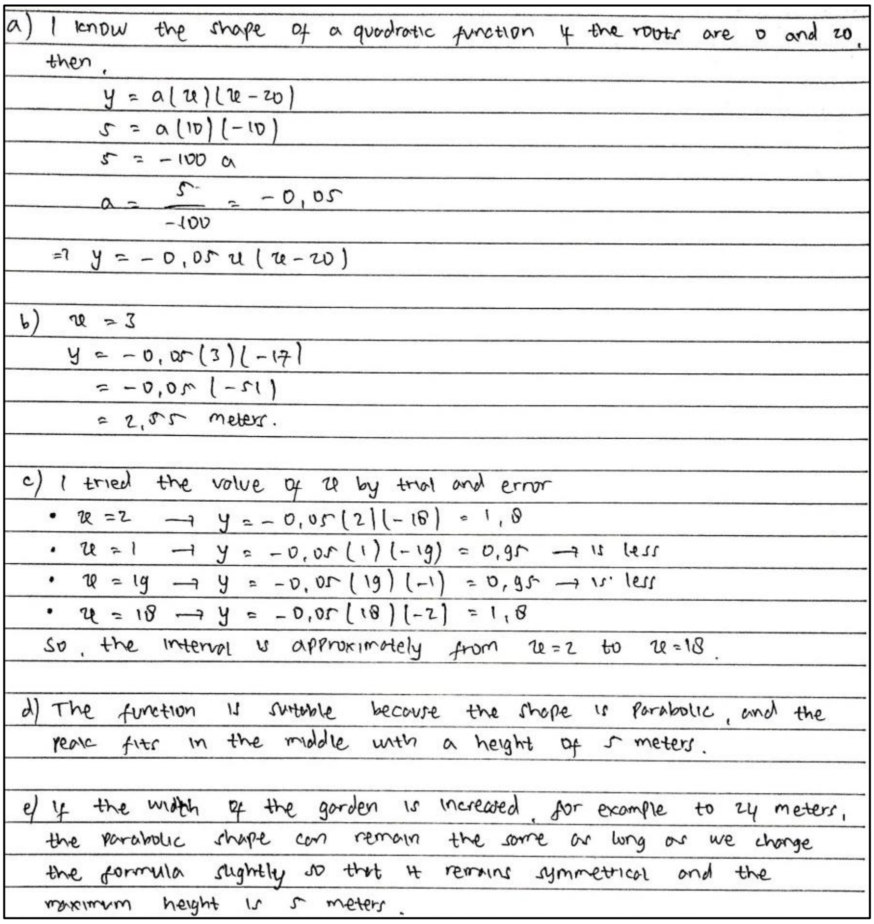
MAS started well in constructing a quadratic function model based on the information that the roots are 0 and 20, as shown in Figure 3. MAS wrote the general form of the quadratic equation from the roots as $y = a(x-0)(x-20)$. Then MAS substituted the vertex (10, 5) to find the value of a. This procedure was done correctly, resulting in:

$$5 = a(10)(-10) \Rightarrow a = -0.05$$

So the function becomes:

$$y = -0.05x(x-20)$$

This step shows that MAS has a fairly good conceptual and representational understanding of quadratic functions.



a) I know the shape of a quadratic function if the roots are 0 and 20, then,

$y = a(x)(x-20)$

$5 = a(10)(-10)$

$5 = -100\,a$

$a = \frac{5}{-100} = -0,05$

$\Rightarrow y = -0,05\,x\,(x-20)$

b) $x = 3$

$y = -0,05(3)(-17)$

$= -0,05\,(-51)$

$= 2,55$ meters.

c) I tried the value of x by trial and error

- $x = 2 \rightarrow y = -0,05(2)(-18) = 1,8$
- $x = 1 \rightarrow y = -0,05(1)(-19) = 0,95 \rightarrow$ is less
- $x = 19 \rightarrow y = -0,05(19)(-1) = 0,95 \rightarrow$ is less
- $x = 18 \rightarrow y = -0,05(18)(-2) = 1,8$

So, the interval is approximately from $x = 2$ to $x = 18$.

d) The function is suitable because the shape is parabolic, and the peak fits in the middle with a height of 5 meters.

e) If the width of the garden is increased, for example to 24 meters, the parabolic shape can remain the same as long as we change the formula slightly so that it remains symmetrical and the maximum height is 5 meters.

Figure 3. MAS's Answer

In part (b), MAS substitutes the value. $x = 3$ into the function to calculate the height of the parabola at that point. The calculation is accurate:

$$y = -0.05(3)(-17) = 2.55 \text{ metres}$$

However, in part (c), MAS used a trial-and-error strategy to find the interval x that yielded a height of approximately 1 metre. Several values were tried, and MAS concluded that the interval was approximately. $x = 2$ to $x = 18$. This strategy is feasible, but the method is inefficient and does not use mathematical rules such as solving quadratic inequalities, which can provide more systematic results. MAS did not solve the inequality $y \leq 1$ algebraically, but merely tried several values. It indicates a lack of algebraic manipulation skills and formal problem-solving abilities that should be utilised to answer questions more efficiently (Alzoebi et al., 2023; Jahudin & Siew, 2023; Newman, 1977).

In stating the final answer in part (c), MAS only mentions the interval from 2 to 18 without stating it in the correct mathematical form, such as:

$$y = -0.05x(x - 20) \leq 1 \Rightarrow x \in [2,18]$$

It indicates that MAS was not thorough in writing down the results in a precise and complete mathematical form, even though the thought process was correct. Parts (d) and (e) demonstrate the students' ability to interpret the graphical meaning of quadratic functions in context, including the reasons for choosing the parabolic model and predictions about changes in the garden's width. These findings indicate reflective ability in contextual mathematical thinking (Setiyani et al., 2022; Yasin et al., 2020).

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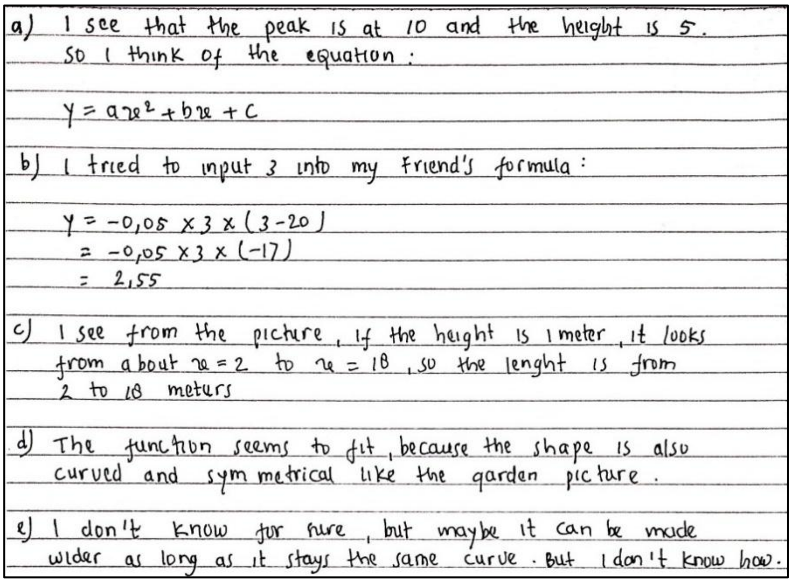
Researcher	:	*In the first part of the question, you wrote the quadratic equation* $y = a(x - 0)(x - 20)$*. How did you know that this was the most appropriate form?*
MAS	:	*Because we know that the intersection points are at 0 and 20, sir. So, I remember from my lessons that if we know the roots, we can directly use the formula* $y = a(x - x_1)(x - x_2)$*.*
Researcher	:	*Good. Then how do you determine the value of a from that function?*
MAS	:	*I see there is a peak at (10, 5), so I put it into the formula. I calculated that* $a = -0.05$*, sir.*
Researcher	:	*For the part that seeks the height of the garden at a position 3 metres from the end, how would you solve it?*
MAS	:	*I substituted* $x = 3$ *into the formula that was created earlier, sir. Then I calculated and obtained 2.55 metres.*
Researcher	:	*In the section where you are asked to find when the height of the garden is less than or equal to 1 metre, you use the trial and error method. Why don't you solve it algebraically?*
MAS	:	*I'm actually confused, sir. I don't remember how to solve quadratic inequalities. So, I tried substituting 1, 2, 3, until the garden height was greater than 1, then I took the difference.*
Researcher	:	*In your opinion, is that method reliable?*
MAS	:	*Yes, maybe you can, sir, but I'm not sure the results will be accurate. It takes a long time to try them one by one, and I'm afraid I'll miss the right number.*
Researcher	:	*Can you explain why you chose a downward-facing parabola?*
MAS	:	*I see that the peak is the highest, but the question says the middle is the highest. So, it must be open at the bottom, sir.*
Researcher	:	*What if the length of the garden is changed to 30 metres? What will happen to the graph and the garden's height?*
MAS	:	*If the length becomes 30, the parabola may widen. The peak point also shifts to the center, so it may be at* $x = 15$*. However, the height may also change, depending on the maximum point.*

Students with medium ability showed adequate mastery of quadratic function representation, value substitution, and contextual understanding. However, they tended to rely on numerical or trial-and-error approaches when confronted with analytical problems involving inequalities. This tendency reflects a partial integration of symbolic and numerical reasoning, a phenomenon often observed among learners in the transitional stage from procedural to conceptual understanding (NCTM, 2000; Polya, 1973). These findings align with the results of the present study, which revealed that many students committed transformation and process-skill errors under the NEA framework. Such errors indicate that MAS students can identify relevant information but still struggle to translate it into appropriate algebraic or symbolic representations. Therefore, instructional strategies that emphasise multiple representations and scaffolding are necessary to strengthen their conceptual connections.

(c) Low Ability Student (LAS)

Figure 4 shows that the vertex of the parabola is at $x = 10$and the maximum height is 5. LAS then writes the general form of the quadratic function $y = ax^2 + bx + c$. However, LAS does not continue determining coefficient values. a, b, and c based on this information. It indicates that the initial understanding of the concept is present, but the mathematical processing is not fully carried out; thus, it is classified as TE.



a) I see that the peak is at 10 and the height is 5. So I think of the equation :

y = ax² + bx + c

b) I tried to input 3 into my friend's formula :

y = -0,05 x 3 x (3-20)
= -0,05 x 3 x (-17)
= 2,55

c) I see from the picture, if the height is 1 meter, it looks from about x = 2 to x = 18, so the lenght is from 2 to 18 meters

d) The function seems to fit, because the shape is also curved and symmetrical like the garden picture.

e) I don't know for sure, but maybe it can be made wider as long as it stays the same curve. But I don't know how.

Figure 4. LAS's Answer

In part (b), LAS only tried to insert the number 3 into the previous formula. LAS did not create the quadratic function formula itself, but only used his friend's formula, namely $y = -0.05x(x - 20)$ and substituted $x = 3$. The substitution was done correctly, even without including units or interpretation. This indicates LAS's inability to process information independently, which is categorised as PE.

Part (c) explains that LAS estimates the range of x values for garden heights ≤ 1 metre to be between $x = 2$ and $x = 18$ based solely on observation of the image without performing algebraic or numerical analysis. It indicates a reliance on visual observation without linking it to mathematical procedures. Sections (d) and (e) reflect on the shape of the parabolic curve. LAS states that the quadratic function is symmetric, but cannot explain why or how changes in the function will affect the shape of the graph. LAS is unable to explain changes in the graph's width, indicating an inability to generalise. This indicates EE.

Researcher	:	*You wrote that the vertex of the parabola is at point 10 and its height is 5, then wrote the general equation $y = ax^2 + bx + c$. Why did you write it in that form?*
LAS	:	*Because I know that's the general form of a quadratic function. If there's a curved graph like a parabola, we usually use that formula, sir.*
Researcher	:	*Are you looking for the values of a, b, and c from that peak information?*
LAS	:	*No, sir. I'm confused about how to do it. I only know the formula, but I don't know how to get the result.*
Researcher	:	*In part (b), you used the formula $y = -0.05x(x - 20)$. Did you come up with that formula yourself?*
LAS	:	*No, it's from my friend, sir. She already made the formula, so I just substituted 3 into it.*
Researcher	:	*Why did you choose to use your friend's formula?*
LAS	:	*The thing is, I can't find the formula myself, sir. So I'm using the existing one to continue with the calculations.*
Researcher	:	*In part (c), you mentioned that the height of the garden is about 1 metre when x is between 2 and 18. Where did you get that from?*
LAS	:	*From the picture, sir. I can see from the shape of the curve. I think the height of 1 metre is somewhere between those numbers.*
Researcher	:	*Are you sure that's accurate?*
LAS	:	*I'm not sure, sir. I'm just guessing from the picture. I don't know how to calculate it.*
Researcher	:	*In parts (d) and (e), you said that the graph is symmetrical and can be made wider. Can you explain what you mean?*
LAS	:	*Hmm...because it's curved and looks like a garden, sir. So, I think it must be symmetrical. If you want to widen it, maybe you can, but I don't know how to change the formula to make the graph wider.*

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Researcher	:	*So you know what the graph looks like, but you don't know how to change the formula to match it?*
LAS	:	*Yes, that's right, sir. I don't understand yet.*

Students categorised as low-ability demonstrated a basic understanding of quadratic functions but were unable to construct equations independently. They tended to rely on memorised formulas rather than reconstructing mathematical relationships from the context. Their intuitive use of visualisation without connecting it to algebraic reasoning reflects limited conceptual coherence. This result aligns with the comprehension and transformation errors identified in the study's data analysis, where LAS frequently misinterpreted problem statements or failed to convert them into mathematical models. Similar findings were reported by Baki & Gürsoy (2020), who noted that low-ability students often exhibit fragmented understanding and low self-efficacy, as also reflected in their verbal responses such as *"I don't know"* or *"I just guessed."* These findings highlight the importance of building students' confidence through guided inquiry and contextualised tasks that gradually develop both conceptual understanding and persistence in problem solving.

4. Conclusion

PE is the most common mistake students make. This shows that the majority of students have difficulty performing mathematical procedures or algorithms correctly, even when they understand the question. Furthermore, CE and TE appear with high frequency, indicating that some students experience obstacles in understanding the question's concept and converting verbal information into the appropriate mathematical form. RE and EE appear in lower proportions but remain significant in influencing the final accuracy of students' answers. These findings indicate that the process of solving mathematical problems is influenced by the ability to read, understand, and interpret contextual problems. The analysis also shows that students' ability levels correlate with the types of errors that appear. HS students make a few errors and can solve problems using the correct procedures. Meanwhile, MAS or LAS students get stuck at the transformation and process stages, leading to incorrect final answers despite partially understanding the problem.

This study had a limited number of subjects, so the results cannot be widely generalised to the entire population of high school students. The analysis used only one type of mathematical reasoning question based on quadratic functions, limiting the diversity of problem contexts analysed. Finally, the error analysis was qualitative, and although interviews were conducted, the subjective data from students may not fully reflect their actual thinking processes. Therefore, it is recommended that future research expand the scope of the study subjects to allow for better generalisation of the results, use a wider variety of questions to uncover student errors more comprehensively, and combine error analysis with the APOS or metacognitive approach to examine the impact of interventions in reducing Newman's types of errors.

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