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Bellman-Ford Algorithm for Shortest Path Selection

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Abstract- Determining the shortest path from one point to another (destination) is a problem often encountered in life. With the development of science and technology, the problem of finding the shortest path can be solved using various algorithms. The algorithm that is often used to find the shortest route is the Bellman-Ford algorithm. The Bellman-Ford algorithm requires initial location parameters and a destination as input to determine the shortest path. The algorithm's results provide the shortest distance along with the route from the initial location to the destination. Applying the Bellman-Ford Algorithm provides the shortest distance as a solution to the problem. This research determines the shortest path from the origin to the destination point, modeled by a weighted graph. The Bellman-Ford algorithm starts the iteration by updating the distance at each vertex if a shorter distance is found.

1. Introduction

With the rapid advancement of digital technology, navigation systems, and route optimization have become increasingly important in various fields of life. The application of shortest path-finding algorithms has found its place in GPS navigation systems, telecommunication network optimization, and many other fields. The ability to determine the shortest path from a starting point to a destination point is a common problem faced by various groups, such as food couriers, firefighters, and computer network designers (Adnan et al., 2020).

Route optimization is important and not just limited to traditional modes of transportation such as land, sea, and air travel. It plays an important role in modern applications such as GPS navigation systems, computer networks, logistics, and city infrastructure planning. As transportation and communication networks become more complex, the demand for efficient and reliable algorithms to determine optimal routes continues to increase. Weighted graphs serve as a useful model to represent the shortest path problem, providing solutions to optimization problems in transportation networks and information systems (Putra et al., 2020).

Among various shortest-path finding algorithms, the Bellman-Ford algorithm is widely studied and

commonly used. It uses a greedy strategy that selects the edge with the smallest weight at each step. Both methods have specific characteristics and advantages that make them suitable for different contexts. The Bellman-Ford algorithm requires parameters that define the origin and destination to find the shortest distance and the associated route (Timofeeva et al., 2023; Zhu et al., 2021).

Based on previous research, Arga et al. (2021) state that the Dijkstra Method determines the shortest path with the minimum cost between one point and another. This research will simulate the search for the shortest path using the Bellman-Ford algorithm to explain how the Bellman-Ford algorithm works in determining the shortest path. (Parimala et al., 2021).

2. Methods

1. Graph

A graph is a structure consisting of vertices (nodes) and edges that connect vertices. Formally, a graph G is defined as an ordered pair $G = (V, E)$, where V is a non-empty set of vertices and E is the set of edges connecting pairs of vertices from V (Buhaerah et al., 2019).

The concepts that exist in graphs are as follows:

1. Node: A point or node in a graph, usually represented as a circle or a dot.
2. Edge: A line connecting two vertices, showing the relationship or connection between the vertices.
3. Degree: The number of edges connected to a vertex.
4. Path: The sequence of vertices and edges connecting one vertex to another.
5. Cycle: A path that starts and ends at the same vertex.
6. Connected graph: A graph with a path between every pair of vertices.
7. Component: A part of the graph that is maximally connected.
8. Weighted Graph: A graph in which each edge is assigned a weight.

Each type of graph has different properties and applications in problem-solving. This study will discuss some main graph types: simple, directed, weighted, and complete. Knowledge of these graph types enriches theoretical understanding and offers practical applications in computer networks, transportation, social analysis, and optimization (MUBARAK, 2019).

Based on the presence or absence of bracelets or double edges in a graph, graphs are grouped into two types, namely:

1) Simple Graph

A simple graph is without double edges and loops, as shown in Figure 1.

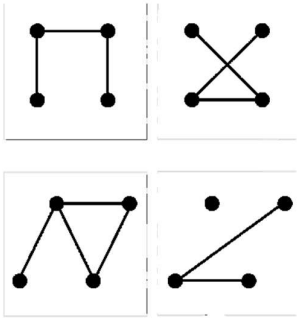

Figure 1. Simple Graph

2) Unsimple Graph

Graphs that contain double edges or rings can be seen in Figure 2.

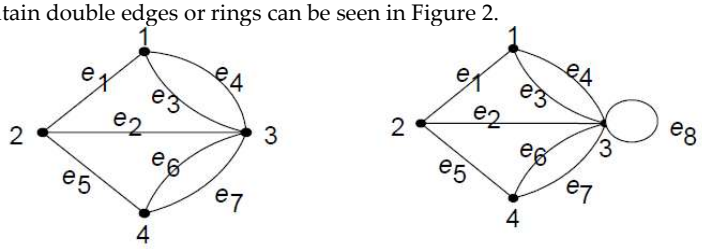

Figure 2. Unsimple Graph

2. Bellman-Ford

American scientists created the Bellman-Ford algorithm as a long-distance vector routing method that

handles the transit section between two network nodes. This algorithm was developed by Richard E. Bellman and Lester Ford, Jr. to find the minimum route of a weighted network structure (Timofeeva et al., 2023). It is similar to Dijkstra's algorithm, but the Bellman-Ford algorithm can handle negative weights in the shortest path finding for weighted graphs (Retanto, 2009).
Bellman-Ford algorithm to determine the shortest path in general. The steps of the Bellman-Ford algorithm are (Sholikah, 2024):

1. Determine the start and end vertices from the list of all vertices and edges.
2. Give the origin node a value equal to 0 and the other nodes an infinite distance value.
3. Start iterating over all vertices, starting from the origin vertex and continuing until all vertices are passed. Calculate the UV = edge connecting U with V, where U is the origin vertex, and V is the destination vertex. If the distance of V is greater than the distance of U + UV weight, then the distance of V is filled with the distance of U + UV weight.
4. Review if there are circuits with negative weights.

3. Shortest Path

A path is a relationship between points or nodes in a graph, while a shortest path is a path that has the minimum total weight to reach a place from a certain place. The shortest paths can be found using graphs. The graph used is a weighted graph, which is a graph in which each edge is given a value or weight. The weight on the graph edge can state time, cost, the distance between cities, places, and so on (Serrano et al., 2019; Afero, 2021).

3) Results and Discussion

Simulation of the shortest path optimization problem-solving in graphs. A connected graph with each vertex as an origin and a destination. The edge in the graph represents the connecting path between one vertex and another, where the edge is given a value called weight. The weight can be in the form of cost, distance, speed, or other parameters. The following explains how the Bellman-Ford Algorithm selects the most dependent path.

1. Shortest path search using the Bellman-Ford algorithm can be seen in Figure 3.

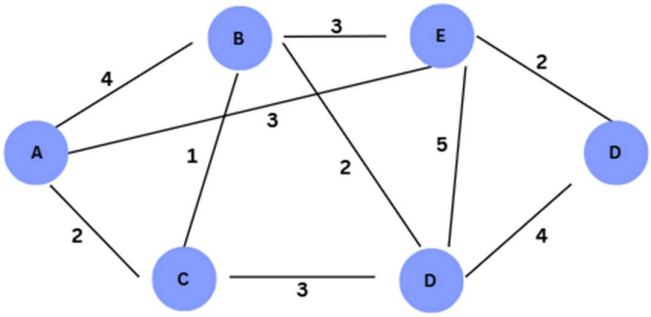

Figure 3. An example of a graph with positive weights

In Figure 3, the results obtained using the Bellman-Ford algorithm are shown in Table 1.

Table 1. Bellman-Ford algorithm search result table

Origin Node	Destination Node	Trajectory
A	B	A-C-B
A	C	A-C
A	D	A-C-D
A	E	A-E
A	F	A-E-F

2. Figure 4 shows the shortest path search using the Bellman-Ford algorithm if there are negative weights.

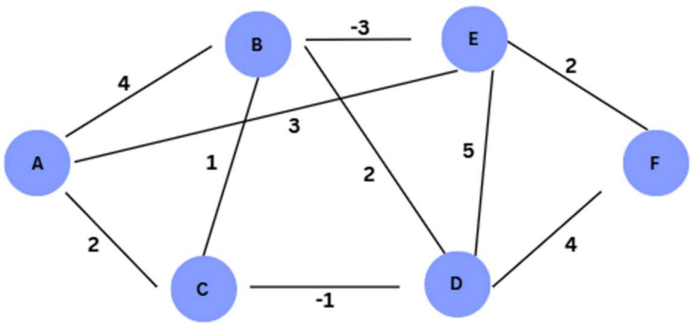

Figure 4. An example of a graph with negative weights

In Figure 4, by using the Bellman-Ford algorithm, results are obtained in Table 2:

Origin Node	Destination Node	Trajectory
A	B	A-C-B
A	C	A-C
A	D	A-C-D
A	E	A-C-B-E
A	F	A-C-B-E-F

4) Conclusion

The results of graph topology using the Bellman-Ford Algorithm to determine the shortest path, namely by checking all paths in the graph that will be passed for each iteration, show that the algorithm can update the distance of each vertex if a shorter path is found and start the iteration by updating the distance at each vertex if the shortest distance is found.

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