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Karush Kuhn-Tucker (KKT) Approach for Production Cost Optimization in Falish Kitchen MSMEs, Prabumulih

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Abstract- In the production process of Micro, Small, and Medium Enterprises (MSMEs), there is often a lack of careful calculation regarding the optimal production quantity for each type of product, which can lead to cost inefficiencies and suboptimal resource utilization. This study aims to minimize production costs at Dapur Falish MSMEs by applying the Karush Kuhn-Tucker (KKT) method as an optimization approach. The optimization model incorporates various constraints, including limited production capacity, to determine the optimal production levels. This problem is modeled mathematically and then formulated in the form of a Lagrange function. Each equation produced is analyzed systematically to ensure that the solution obtained meets the optimal conditions based on KKT requirements. The results show that the minimum production cost for Dapur Falish is IDR 897,389.7 per month. These findings provide practical insights for MSME owners in improving cost efficiency and support more effective production planning by identifying optimal resource allocation under existing constraints.

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1. Introduction

Optimization is an important branch of applied mathematics that aims to minimize costs in order to maximize profits in solving a problem (R. N. Sari, 2024). In the business world, entrepreneurs strive to optimize profits through capital cost efficiency without reducing the quality of products offered to consumers. A business company can formulate an optimal production strategy through a weighted multi objective nonlinear objective function approach, which not only maximizes the selling price but also allows efficient production cost control (Inayati & Rahmawati, 2020). In practice, business actors are often faced with various challenges such as high competition and low consumer interest in the products offered (Putri et al., 2024). These conditions

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directly affect the potential profits that can be achieved by the company, so the best decisions are needed to overcome the sales problems faced.

A similar problem is also faced by one of the culinary businesses from Prabumulih City, namely Dapur Falish. The main products of this MSME are bento, pempek, and cakes. The problem currently faced by this MSME is the limited supply of raw materials, which has an impact on its inability to optimally allocate resources to produce all available product variants. This condition hinders the achievement of maximum profits, so it is necessary to apply optimization methods as a systematic approach to decision making (Rahman et al., 2021). The decisions implemented are expected to encourage increased efficiency and profitability of the business. One mathematical approach that can be applied in the product optimization process is the Karush Kuhn-Tucker (KKT) method.

The KKT method is an optimization technique used to solve both maximization and minimization problems subject to inequality constraints (Ahmad et al., 2024). In its formulation, the method incorporates Lagrange multipliers (λ) as essential components in establishing the optimality conditions (Mardiyanti et al., 2021). These multipliers play a key role in linking the objective function with its constraints to identify optimal solutions (Agu et al., 2024). In general, the KKT conditions provide a fundamental and widely applicable framework for determining optimal points in nonlinear optimization problems with complex constraints (Abo-Sinna & Ragab, 2023).

Previous studies have applied the KKT method in various optimization problems. Research by (Mudhi Zayyan & Isro'il, 2023) used the KKT method in a study titled "Optimization of Production Output $x_j \geq 0 \, ; x_j \in \mathbb{Z}$ Using the Kuhn-Tucker Method." Similarly, research by (W. K. Sari et al., 2025) used the KKT method in its study "Profit Optimization in Kimbab Production with Karush Kuhn Tucker Modeling". These studies demonstrate the effectiveness of the KKT approach in solving constrained optimization problems in production contexts.

However, most existing studies primarily focus on maximizing output or profit and tend to use simplified of theoretical models, with limited consideration of real operational conditions in MSMEs (Rusdiana et al., 2024). In particular, the integration of data-driven objective functions, such as those derived from regression models, and their application to cost minimization problems in small-scale businesses remain underexplored (Rosmasari & Jatiningrum, 2021). Therefore, this study aims to fill this gap by applying the KKT method to optimize production costs in Dapur Falish MSMEs using a polynomial regression-based objective function and real operational constraints. The novelty of this study lies in combining empirical modeling with the KKT framework to provide a more practical and applicable approach for production planning and cost efficiency in MSMEs.

2. Methods

This research was conducted using a literature review approach by collecting data and information from various sources, including books and scientific journals. In addition, data were collected through direct interviews with the relevant parties. The KKT method was used as an analytical approach in this study because of its effectiveness in solving optimization problems accompanied by constraints.

(a) Linear Integer Programming

Integer programming is a special form of linear programming applied to solve optimization problems. This method requires all decision variables to have integer values. According to (W. K. Sari et al., 2025), integer linear programming has the following general form:

$$Max/Min\ Z = \sum_{j=1}^{n} c_j x_j$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}x_j(\leq/=/\geq)b_1 \; ; i = 1,2,3,\dots,m$$
$$x_j \geq 0 \; ; x_j \in \mathbb{Z}$$

Where:

Z : objective function

c_j : contribution coefficient of each decision variable x_j to the increase value of Z

x_j : decision variable j with $= 1,2,\dots,n$

a_{ij} : the amount of resource i required to produce activity j per unit

b_i : the capacity of available resources that can be allocated to each activity

(b) Lagrange Multiplier Method

The Lagrange multiplier method is used to find solutions to optimization problems that use multiple variables and equation constraints. In solving optimization problems with equation constraints, the Lagrange function can be used. According to (Ghojogh et al., 2021) The Lagrange function can be written as follows:

$$L(x,\lambda) = f(x) + \sum_{j=1}^{m} \lambda_j g_j(x)$$

If there is an optimization problem with one constraint, the Lagrange function can be written as follows:

$$L(x,\lambda) = f(x) + \lambda(g(x) - b)$$

This means that the partial derivatives of L with respect to the decision variable x and the multiplier λ are calculated to find the optimal value. This equation is the source for determining the optimal values for x and λ that provide the best solution to the optimization problem with constraints, so that:

$$\frac{\partial L}{\partial x_i} = 0 \; ; i = 1,2,\dots m$$
$$\frac{\partial L}{\partial \lambda} = 0$$

(c) Karush Kuhn – Tucker Method

The KKT method is an extension of the Lagrange method designed to solve both linear and nonlinear optimization problems with constraints in the form of equations or inequalities. According to (Ningrum et al., 2023), there is a definition of the KKT method:
Suppose we are given an optimization problem with constraints:

Maximize/Minimize : $Z = f(X)$ with $X = (x_1, x_2, x_3, \dots, x_n)^t$

Subject to : $g_j(X) \leq 0$ or $g_j(X) \geq 0$

$x_i = 1,2,3,\dots,n$

To solve this optimization problem, the following definition is used:

Definition 1. *Given the optimality conditions of the Lagrange formulation under a set of constraints, let* $X = (x_1, x_2, x_3, \dots, x_n)$ *be the optimal solution for the objective function* $f(x)$ *subject to the constraint functions* $g_j(x)$. *Then there are exist Lagrange multipliers* $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m$ *such that the following conditions are satisfied:*

$$g_j(X) \leq 0 \qquad j = 1,2,3,\dots,m$$
$$\lambda_j[g_j(X)] = 0 \qquad j = 1,2,3,\dots,m$$
$$\lambda_j = 0 \qquad j = 1,2,3,\dots,m$$

Where $L(X,\lambda) = f(X) + \sum_{j=1}^{m} \lambda_j g_j(X)$

Then there is a condition according to (Sinha et al., 2019) where:

$$\frac{\partial L}{\partial x_i} = 0, (i = 1,2,3,\dots,m)$$

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$$\frac{\partial L}{\partial \lambda_j} = 0, (j = 1,2,3, \dots, m)$$

Based on Definition 1, the conditions for KKT in this case are obtained in Table 1.

Table 1. Sufficient conditions for KKT

Type of Optimization	Condition on		
	$f(x)$	$g_j(x)$	λ_j
Maximization	Concave	Convex	$\lambda_j \geq 0$ $(1,2,3, \dots m)$
		Concave	$\lambda_j \leq 0$ $(m+1, \dots, n)$
		Linear	No restriction for $(i = n+1, \dots, p)$
Minimization	Convex	Convex	$\lambda_j \geq 0$ $(1,2,3, \dots m)$
		Concave	$\lambda_j \leq 0$ $(m+1, \dots, n)$
		Linear	No restriction for $(i = n+1, \dots, p)$

3. Results and Discussion

This study used primary data obtained directly from Dapur Falish MSMEs from June 2025 to October 2025. The data collected includes production costs, the number of products produced per month, and raw material constraints. The products offered by Dapur Falish consist of bento, pempek, and cakes. The minimum production for these products is 50, 100, and 30 products, respectively. The objective function in this study is formulated based on production volume and production costs. Production volume is obtained for each period, while production costs are calculated based on monthly expenses. Cost estimates for each product type are determined using a polynomial regression approach analyzed using the KKT method.

The objective function to be achieved is to minimize production costs. Polynomial regression in the formation of the objective function is formed using MATLAB (Matrix Laboratory) software through the fitpoly command. The objective function obtained is as follows:

$$f(x_1, x_2, x_3) = 19.728x_1^2 + 15.63x_2^2 + 59.524x_3^2 + 8921.275x_1 + 2450.967x_2 + 5119.048x_3 - 206533.745 \quad (1)$$

The constraint function in this problem is the minimum production quantity for each product, so that a constraint function can be formed:

$$\begin{aligned} x_1 &\geq 50 \\ x_2 &\geq 100 \\ x_3 &\geq 30 \end{aligned} \quad (2)$$

The next step is to form the Lagrange function to determine the critical points. Equations (1) and (2) can be transformed into the Lagrange function form as follows:

$$\begin{aligned} L(X,\lambda) &= f(X) + \textstyle\sum_{j=1}^{m} \lambda_j g_j(X) \\ &= 19.728x_1^2 + 15.63x_2^2 + 59.524x_3^2 + 8921.275x_1 + 2450.967x_2 + 5119.048x_3 - 206533.745 + \\ &\quad \lambda_1(50 - x_1) + \lambda_2(100 - x_2) + \lambda_3(30 - x_3) \\ &= 19.728x_1^2 + 15.63x_2^2 + 59.524x_3^2 + (8921.275 - \lambda_1)x_1 + (2450.967 - \lambda_2)x_2 + (5119.048 - \\ &\quad \lambda_3)x_3 - 206533.745 + 50\lambda_1 + 100\lambda_2 + 30\lambda_3 \end{aligned} \quad (3)$$

Equation (3) is a Lagrange function and will be used as an objective function in an effort to minimize production costs. The function is then transformed into the general form of the KKT condition as follows:

$$\frac{\partial L}{\partial x_1} = 39.456x_1 + 8921.275 - \lambda_1 = 0 \quad (4)$$

$$\frac{\partial L}{\partial x_2} = 31.26x_2 + 2450.967 - \lambda_2 = 0 \quad (5)$$

$$\frac{\partial L}{\partial x_3} = 119.048x_3 + 5119.048 - \lambda_3 = 0 \quad (6)$$

$$\frac{\partial L}{\partial \lambda_1} = 50 - x_1 = 0 \quad (7)$$

$$\frac{\partial L}{\partial \lambda_2} = 100 - x_2 = 0 \quad (8)$$
$$\frac{\partial L}{\partial \lambda_3} = 30 - x_3 = 0 \quad (9)$$

$x_j \geq 0$ for each $j = 1, 2, 3$
$\lambda_j \geq 0$ for each $j = 1, 2, 3$

Through equations (7), (8), (9), the following values are obtained:
$x_1 = 50$
$x_2 = 100$
$x_3 = 30$

Next, substitute the value of x_1 into equation (4) to obtain the value of λ_1.
$39.456x_1 + 8921.275 - \lambda_1 = 0$
$\Leftrightarrow 39.456(50) + 8921.275 - \lambda_1 = 0$
$1972.8 + 8921.275 - \lambda_1 = 0$
$\lambda_1 = 10894.075$

Next, substitute the value of x_2 into equation (5) to obtain the value of λ_2.
$31.26x_2 + 2450.967 - \lambda_2 = 0$
$\Leftrightarrow 31.26(100) + 2450.967 - \lambda_2 = 0$
$5575.967 + 2450.967 - \lambda_2 = 0$
$\lambda_2 = 5576.967$

Last, substitute the value of x_3 into equation (6) to obtain the value of λ_3.
$119.048x_3 + 5119.048 - \lambda_3 = 0$
$\Leftrightarrow 119.048(30) + 5119.048 - \lambda_3 = 0$
$3571.44 + 5119.048 - \lambda_3 = 0$
$\lambda_3 = 8690.488$

Based on the calculation results of the equation system that has been compiled, the values of $x_1, x_2,$, and x_3 are obtained, which are 50, 100, and 30, respectively. The Lagrange multipliers λ_1, λ_2, and λ_3 are also obtained, which are 10894.075, 5576.967, and 8690.488, respectively. The total production cost is determined using the Lagrange function, which aims to obtain the minimum value of the total cost.

$$\begin{aligned}(X, \lambda) &= f(X) + \textstyle\sum_{j=1}^{m} \lambda_j g_j(X) \\ &= 19.728x_1^2 + 15.63x_2^2 + 59.524x_3^2 + (8921.275 - \lambda_1)x_1 + (2450.967 - \lambda_2)x_2 + (5119.048 - \lambda_3)x_3 - 206533.745 + 50\lambda_1 + 100\lambda_2 + 30\lambda_3 \\ &= 19{,}728(50)^2 + 15.63(100)^2 + 59.524(30)^2 + (8921.275 - 10894.075)50 + (2450.967 - 5576.967)100 + (5119.048 - 8690.488)30 - 206533.745 + 50(10894.075) + 100(5576.967) + 30(8690.488) \\ L(X, \lambda) &= 897389.7\end{aligned}$$

Based on the results of the analysis, the estimated minimum production cost required to run Dapur Falish's production is IDR 897,389.7 per month. The minimum production quantities for bento (x_1), pempek (x_2), and cakes (x_3) are 50, 100, and 30 products, respectively. These results reflect an optimal allocation of production resources under the given constraints, where each product level is determined not only by cost considerations but also by feasibility of operational limitations.

These results indicate that pempek plays a more dominant role in the optima production plan due to its higher production quantity, suggesting a relatively more efficient contribution to the overall cost structure. In contrast, bento and cakes are produces in lower quantities, which reflects their constrained contribution within the optimization framework and the need to balance limited production capacity across all products.

Furthermore, the optimization results demonstrate that the combination of production quantities is not determined solely by demand but also by operational constraints such as production capacity and cost structure. This emphasizes the relevance of applying a mathematical optimization approach in MSMEs

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to support more structured, efficient, and data-driven production planning decisions.

4. Conclusion

This study has implemented the KKT method in optimizing the Dapur Falish MSME in Prabumulih City. With a polynomial regression-based objective function approach, the formulation results show that the objective function structure and constraints designed are able to represent operational conditions. Through the application of the KKT method, an optimal solution was obtained with a minimum cost estimate of IDR 897,389.3 per month, highlighting the potential of the KTT method in improving cost efficiency and resource allocation in MSMEs. The main contribution of this study lies in application of a mathematical optimization framework that combines regression modeling and KTT conditions to support practical production planning in small-scale businesses. For future research, it is recommended to incorporate additional real-world factors such as demand uncertainty, dynamic pricing, and multi-period production planning to enhance the model's applicability. Further study may also explore the integration of other optimization techniques or hybrid methods to compare performance and improve solution robustness.

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