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Exploring Students' Mathematical Representation through the Lens of APOS Theory of APOS Theory: An Exploratory Study

Aqilul Asra[1], and Gumpanat Boriboon[2]

[1] Universitas Negeri Padang
[2] Srinakharinwirot University

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Author for correspondence:
Aqilul Asra
Email: aqilulasra@gmail.com

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Abstract- Mathematical representation ability is one of the key competencies that reflects the depth of students' conceptual understanding in mathematics learning. However, various studies show that mathematics education students still face difficulties in using and connecting multiple forms of representation, leading to limited conceptual understanding. This study aims to analyse students' mathematical representation abilities using the APOS (Action, Process, Object, Schema) theoretical framework to reveal the underlying mental mechanisms. A qualitative descriptive method with data triangulation (test and interview) was employed. Six mathematics education students were selected and categorized into low (score < mean – SD), medium (mean – SD ≤ score < mean + SD), and high (score ≥ mean + SD) ability groups based on their mathematical representation test results. Data from tests and interviews were analysed through qualitative coding to ensure reliability and credibility. The findings indicate that low-ability students tended to remain at the Action stage, medium-ability students reached the Process stage, and high-ability students began to reach the Object and Schema stages. This study confirms that the quality of mathematical representation is closely related to students' cognitive stages according to the APOS theory and introduces a novel link between representation indicators and APOS stages, offering valuable insights for mathematics education research.

1. Introduction

Mathematical representation is one of the important competencies that characterises students' conceptual understanding in mathematics learning. In communicating ideas, external representationsd are needed in the form of actions (contextual), verbal, symbolic, visual, and real objects (physical) (NCTM, 2000; Umbara et al., 2019). Representations serve to: (1) inform teachers about how students think about mathematical content or ideas, (2) provide information about student patterns and trends, and (3) act as tools in the learning process. Student representations are not only useful for describing and communicating mathematical objects but also for working with mathematics, such as solving problems in mathematics and with mathematic (Wulandari et al., 2019). When students construct knowledge, it is important to know whether they can represent it to determine how much knowledge they have.

Although important, a number of studies show that mathematics education students still face obstacles in using representations meaningfully. This is based on research (Yudhanegara et al., 2014) showing that students experience difficulties in representing mathematical expressions, namely creating images of equations or mathematical models from other given representations. Another study (Amaliyah AR & Mahmud, 2018) analysing students' problem-solving and mathematical representation abilities shows that the average achievement in solving geometry problems is in the moderate category at 64.81%.). Ikashaum (2021) also found errors made by students in creating symbolic representations in geometry problems, namely a lack of understanding of concepts, errors in reading the data contained in the images, and errors in performing mathematical operations. This condition implies a weak conceptual understanding because students tend to work algorithmically. This was also confirmed by Duval (2017), who stated that the main difficulty for students was not only in mastering one type of representation but also in the ability to translate between representations. In classroom practice, such difficulties often appear when students can compute algebraic solutions but fail to connect them with graphical or geometric representations. For instance, many students can correctly manipulate equations yet are unable to explain what the slope or area in a diagram represents, or they misinterpret three-dimensional geometric problems when translating them into symbolic form. These patterns indicate that students' reasoning remains procedural and fragmented across representation types, limiting their ability to form integrated conceptual understanding.

To understand how students construct mathematical representations, a cognitive theoretical framework is needed to describe their mental mechanisms. One theory widely used in mathematics education is the APOS Theory (Action, Process, Object, Schema) developed by Arnon et al., (2014) and Dubinsky & McDonald (2001). This theory explains how a mathematical concept is constructed through mental stages: from procedural actions (Action), internalisation of processes (Process), understanding concepts as objects (Object), to the formation of a complete knowledge structure (Schema)(Arnon, Cottrill, Dubinsky, et al., 2014; Dubinsky & McDonald, 2001; Fitria et al., 2024; Martínez-Planell & Cruz Delgado, 2016; Orozco-santiago & Trigueros, 2008; Syamsuri & Marethi, 2018). The APOS theory has been applied in understanding and misconceptions in various areas of mathematics. For example, this theory has been used to study the learning of binomial expansion (Tatira, 2021; Tatira & Mukuka, 2024), exponential and logarithmic functions (Díaz-Berrios & Martínez-Planell, 2022; Okoye-Ogbalu & Nnadozie, 2023), calculus (Borji et al., 2018; Siyepu, 2013), trigonometry (Martínez-Planell & Cruz Delgado, 2016; Nabie et al., 2018; Padma Mike Putri M & Martin, 2024), geometry (Wulandari et al., 2019) and linear algebra (Mutambara & Bansilal, 2019).

APOS is particularly suitable for analyzing mathematical representations because various mathematics education research scenarios, providing insights into students' each stage corresponds to different ways students express and connect mathematical ideas through various representational forms. At the Action stage, students tend to rely on external and procedural representations, such as symbolic manipulation or drawing figures mechanically, without deep conceptual links. The Process stage involves the internalization of these actions, where students begin to mentally operate on representations, for instance by interpreting how an algebraic formula corresponds to a graph or diagram. The Object stage marks the point when students can treat a representation as a coherent mathematical entity—such as recognizing a function graph not merely as a picture but as an object with properties and relationships. Finally, at the Schema stage, students integrate multiple representations (verbal, symbolic, graphical, and contextual) into a unified conceptual framework, enabling them to flexibly translate between them when solving problems.

However, studies that integrate the analysis of mathematical representation abilities with the APOS

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framework are still limited. Most previous studies have only focused on descriptive analysis of representation without mapping the underlying mental mechanisms. Conversely, APOS research has mostly highlighted the construction of mathematical concepts, but has not been specifically used to analyze how students construct and translate representations. This gap is the urgency of this study. Integrating APOS theory with representation analysis is expected to provide deeper insights into students' cognitive development and how they transition from procedural to conceptual understanding. Such integration may also inform the design of instructional strategies that explicitly foster multi-representational thinking and support more effective learning progression.

Based on this gap, this article presents a novel approach by analyzing students' mathematical representation abilities through the perspective of APOS theory. This analysis not only describes students' achievements in various types of representations but also explores their cognitive stages (Action, Process, Object, Schema) in constructing representations. Thus, this study is expected to contribute theoretically by strengthening the role of APOS in the study of mathematical representations, as well as contributing practically by providing considerations in choosing learning strategies that can encourage students to transition towards a deeper conceptual understanding.

2. Methods

(a) Research Design

This study is a descriptive qualitative study. This study describes how representations are used by students in constructing geometry. This study explores the thinking process in constructing evidence(Creswell & Creswell, 2018; Harisman et al., 2025). This study uses a qualitative approach for three reasons, namely: (1) the researcher as a key instrument, (2) inductive data analysis, and (3) holistic explanation.

(b) Participant

The analysis focused on identifying students' cognitive stages (Action, Process, Object, Schema) and mapping them to their use of mathematical representations.This study was conducted at a state university in Padang province and involved 19 mathematics education students who had studied the subject of Plane and Solid Geometry. Furthermore, from the 19 students, 6 students will be selected, each of whom will represent high (score ≥ mean + SD), low (score < mean – SD), and moderate (mean – SD ≤ score < mean + SD) abilities for further interviews (Ebel & Friesbie, 1991). The six students were coded G1, G2, G3, G4, G5, and G6 to facilitate the analysis process.

(c) Instrument

Data were analysed using qualitative descriptive techniques involving data reduction, data display, and conclusion drawing as proposed by Creswell & Creswell (2018). The instrument used in this study was a mathematical representation test in the field of geometry consisting of 5 questions, which can be seen in the appendix. The questions were answered by the students, and the answers were analysed to trace how the students' representation process was carried out based on the APOS theory. The students' work will be reinforced through interviews conducted after the students have completed the test questions below. Thus, the essence of the proof task and interview transcripts will be obtained. The representation indicators used are presented in Table 1.

Table 1. Representation Ability Indicators

No	Indicators	Expected achievement
1	Organising, recording, and communicating mathematical ideas	1. Students are able to write down solutions in the form of tables, diagrams, graphs, pictures, equations, or words. 2. Students can explain mathematical ideas through the chosen representation.

No	Indicators	Expected achievement
2	Selecting, applying, and translating between different mathematical representations to solve problems.	1. Students can move from verbal to symbolic representations, from tables to graphs, or from pictures to equations.. 2. Students are able to use more than one type of representation to explain the solution to a problem.
3	Using representations to model and interpret physical, social, and mathematical phenomena.	1. Students are able to use graphs or models to understand and analyse real-life situations. 2. Students can relate contextual problems to mathematical forms (for example, formulating equations from everyday problems).

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3. Results

(a) Mathematical Representation Test Results

Based on the results of the mathematical representation ability test, data was obtained on the scores of 19 students, with a scoring scale of 0-3 for each question, where 0 means no answer was given, 1 means an answer was given but was incorrect, 2 means an answer was given that was partially correct, and 3 means the answer was correct. The data on the results of the students' mathematical representation ability test based on the average per indicator can be seen in Table 2.

Table 2. Mathematical Representation Ability Test Results

No	Representation Indicator	Average Score
1	Organising, recording, and communicating mathematical ideas	1.71
2	Selecting, applying, and translating between different mathematical representations to solve problems	1.76
3	Using representations to model and interpret physical, social, and mathematical phenomena.	1.35

Table 2 shows that students' representation skills are still quite low, as can be seen from the average student scores for each indicator, which are still far from the perfect score of 3. The highest average achievement of students is in indicator 2, which is 1.76. Meanwhile, the lowest representation ability is found in indicator 3, which is using representations to model and interpret physical, social, and mathematical phenomena, with an average score of 1.35.

In addition to describing students' mathematical representation abilities in general, The test results were also used as a basis for selecting students with high, medium, and low abilities, namely G1, G2, G3, G4, G5, and G6. The six selected students were interviewed to explore their representation abilities in depth based on the APOS theory.

(b) Analysis of Mathematical Representation Abilities with the APOS Theory

Based on the student test results and interview results, data on the level of students' representation abilities based on the APOS theory was obtained from six students (G1, G2, G3, G4, G5, and G6). An overview of student abilities based on the APOS theory for each indicator can be seen in Table 3.

Table 3. Students' Mathematical Representation Ability Based on APOS Theory

Students	Representation Indicators		
	1	2	3
G1	Scheme	Scheme	Processes
G2	Objects	Processes	Actions
G3	Scheme	Processes	Actions
G4	Processes	Processes	Processes
G5	Actions	Actions	No Respon
G6	Actions	No respon	Actions

The following is a description of the research results based on the researcher's analysis in accordance with the representation indicators used.

1. Organising, recording, and communicating mathematical ideas

In the first indicator, there are four stages of mathematical ability demonstrated by students. The action stage appeared in students G5 and G6. The students' answers at the action stage can be seen in Figure 1.

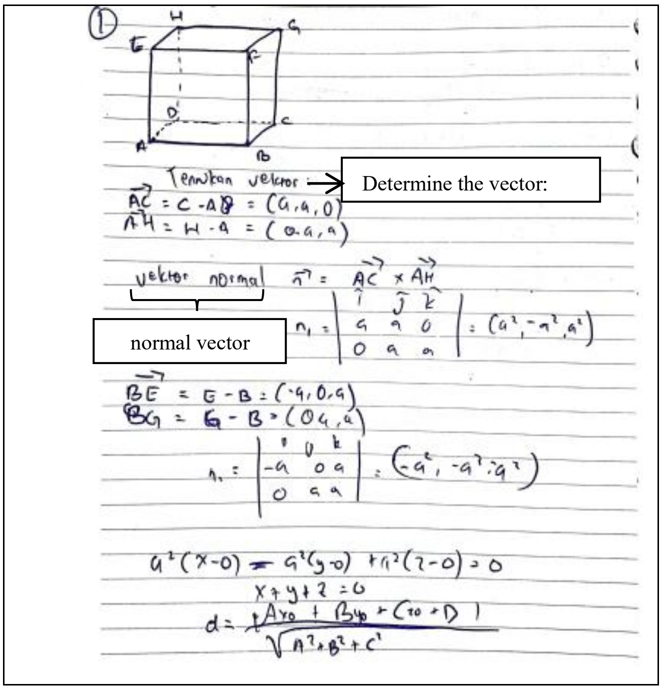

Figure 1. Answer G5 for question in indicator 1

In this section, G5 uses algebraic principles, but does not understand what he is doing. When asked why he is looking for vectors AC and AH, G5 cannot explain. G5 can only copy what he vaguely remembers. This proves that G5 has only reached stage 1 of the indicators. Meanwhile, students who have reached the process stage can be seen in Figure 2.

Translation :
Distance between plane ACH and plane BEG?
Answer: P = plane ACH intersects diagonal DF
Q = plane BEG intersects diagonal DF

$$DP = \frac{1}{3} DF$$
$$FQ = \frac{1}{3} DF$$
$$DF = a\sqrt{3}$$
$$Therefore: DP = \frac{a\sqrt{3}}{3}\ cm$$
$$FQ = \frac{a\sqrt{3}}{3}\ cm$$

The distance between plane ACH and plane BEG is PQ
$$\mathrm{PQ} = \mathrm{DF} - \mathrm{OP} - \mathrm{FQ}$$
$$\mathrm{PQ} = a\sqrt{3} - \frac{a\sqrt{3}}{3} - \frac{a\sqrt{3}}{3}$$
$$\mathrm{PQ} = \frac{a\sqrt{3}}{3}$$

Figure 2. Answer G4 for question in indicator 1

In Figure 2, it can be seen that G4 was able to identify the cube and the planes ACH and BEG. G4 also wrote down the line intersecting planes ACH and BEG, namely diagonal DF. G4 identifies that $DP = FQ = \frac{1}{3}\ DF$ DF and $DF = a\sqrt{3}$ and calculates that $PQ = \frac{a}{3}\sqrt{3}$ When asked why $DP = FQ = \frac{1}{3}$DF, G4 states that they know that the length is $1/3$ of the length of the space diagonal, which they learned at school. However, G4 cannot prove this through algebraic or geometric representation. From the answer, G4 can already imagine and determine the line used to calculate the distance between planes ACH and BEG. This indicates that G4 did not just draw and copy the question but was able to visualise the distance between planes ACH and BEG. However, G4 did not manipulate it algebraically to determine the exact distance between the two planes, so G4 reached the Process stage. Meanwhile, students who reached the Object stage can be seen from G2's answer in Figure 3.

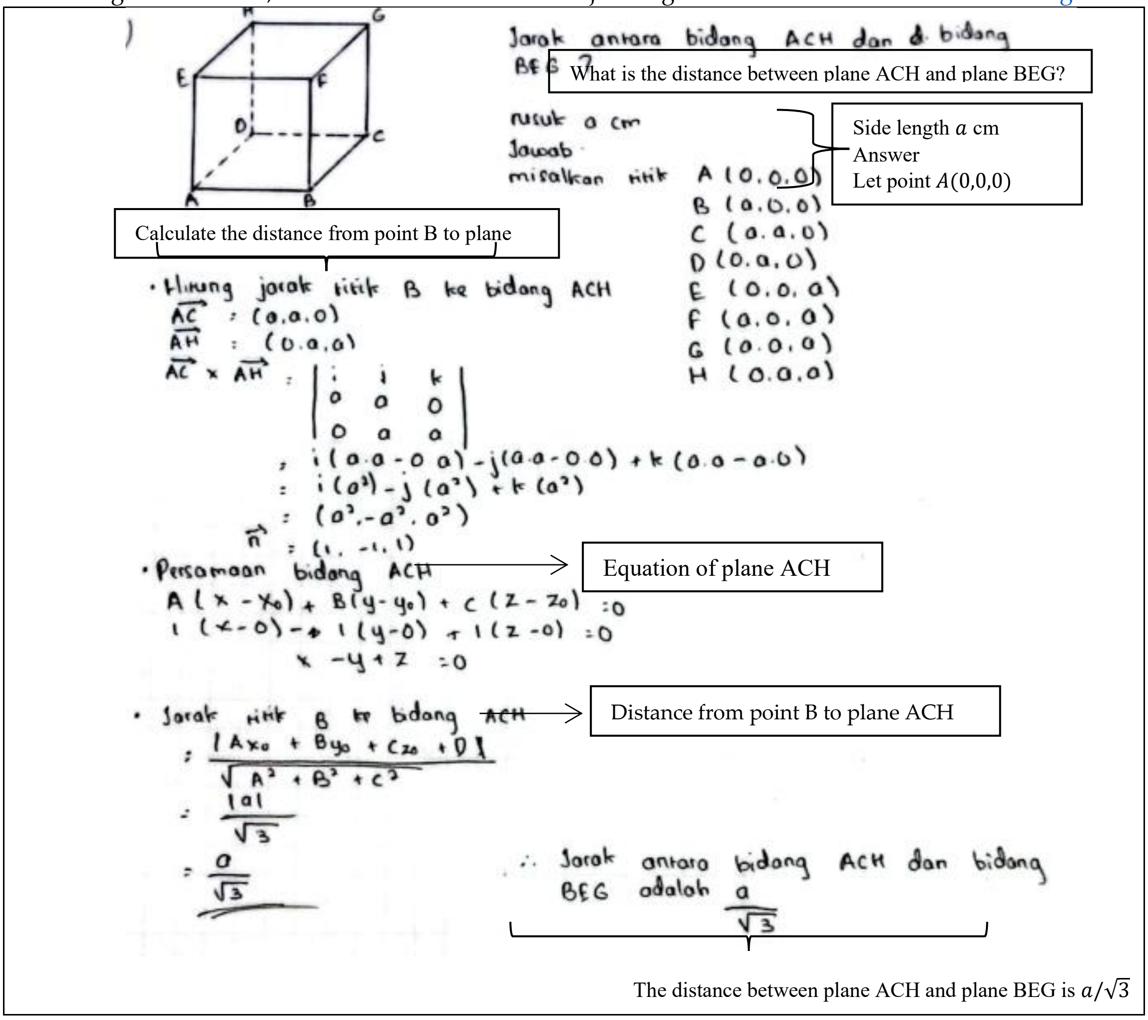

Figure 3. Answer G2 for question in indicator 1

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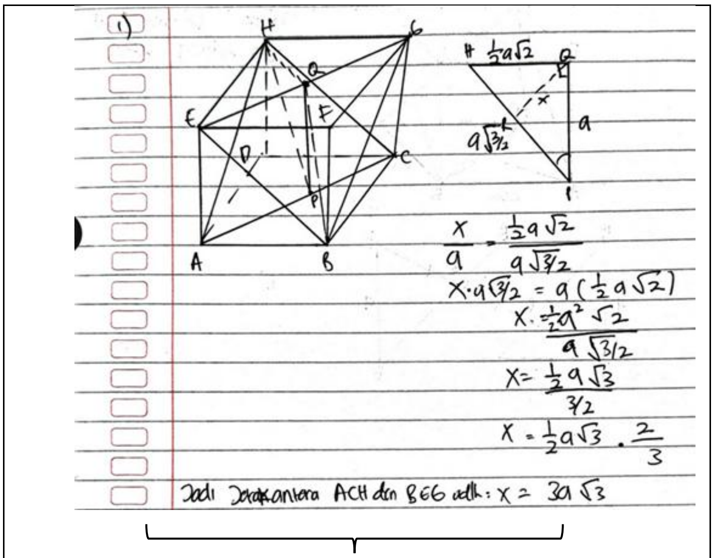
For G2's answer to indicator 1, it can be seen that G2 can identify what is known from the question by drawing a picture and also by assuming coordinate points with $A(0,0,0), B(a,0,0), \ldots, H(a,a,a)$. Next, G2 determines the equation of the ACH plane and uses it to determine the distance between point B and the ACH plane, which is connected by a perpendicular line to the ACH plane. However, when asked to explain, G2 only rereads what they have written and is unable to connect the algebraic representation they have written with the given graphic representation. G2 also still relied on the formula without understanding its use in depth, even though G2 was able to write the equation and manipulate it to determine the distance between planes ACH and BEG. Therefore, G2 had reached the object stage. Next, for students who had reached the schema stage, the answers can be seen in Figure 4.

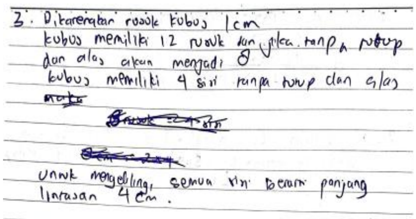


Therefore, the distance between ACH and BEG is $3a\sqrt{3}$

Figure 4. Answer G1 for question in indicator 1

G1 correctly represented the distance between planes ACH and BEG by drawing an auxiliary line between the midpoint of line EG, point Q, and the midpoint of line AC, point P. G1 then connected points Q and P. Next, G1 drew line HP, forming triangle HPQ. Next, to make it easier to calculate the distance between the two planes, G1 represents triangle HPQ outside the cube $ABCD\ EFGH$. Then G1 draws a line from point Q to point R, which is the midpoint of line HP, where line QR, symbolised by G1 with x, is the distance between the two planes asked in the problem. G1 finds the length x using the sine ratio of triangles HPQ and HPX, but G1 makes a mistake in the algebraic calculation and quickly realises this when asked. In this case, G1 has shown that he is able to think from the action-object stage and connect the various representations well in writing and verbally, so that G1 has reached the Schema stage in indicator 1.

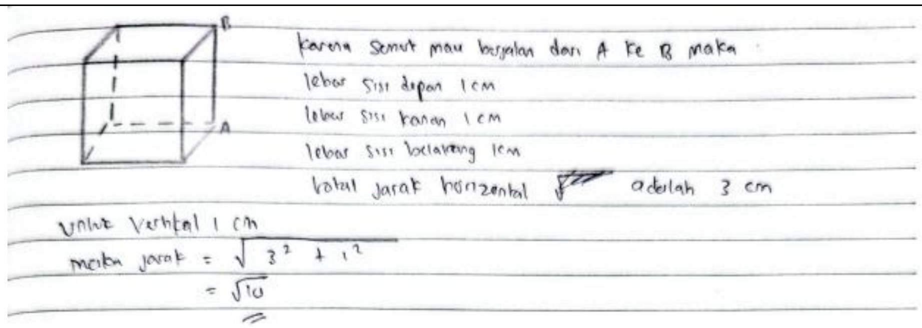
2. Selecting, applying, and translating between various mathematical representations to solve problems
   For the second indicator, students were found to have achieved 3 stages, namely actions, process, and scheme. The student who could not be identified using the APOS theory was G6 because he did not respond to the questions in the problem. When asked to explain or think of an idea for the problem, G6 was still unable to answer the questions. Meanwhile, G5 was able to reach the action stage, and G5's answer to indicator 2 can be seen in Figure 5.



Translate :
Because the edge of the cube is 1 cm
The cube has 12 edges, and if there is no lid or base, there will be 8
The cube has 4 sides without a lid or base
To go around all sides, the path length is 4 cm

Figure 5. Answer G5 for question in indicator 2

To answer the question in indicator 2, G5 wrote down all the important information he obtained from the question and concluded that to go around all sides, the length of the path would be 4 cm. From this answer, G5 was not yet able to choose a representation and imagine the process or solution in solving this mathematical problem. When explaining their answer, they reiterate what they have written. Here, G5 can only copy what is known, which identifies that G5 has the ability to reach the action stage for the second indicator. Furthermore, for students who have reached the process stage of thinking, their answers can be seen in Figure 6.

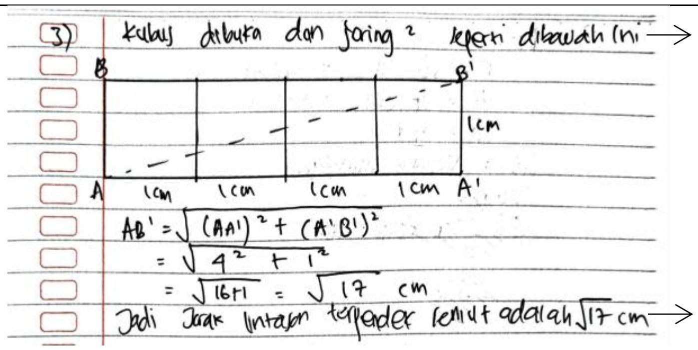


Translation :

Because the ant wants to walk from A to B,
the width of the front side is 1 cm
the width of the right side is 1 cm
the width of the back side is 1 cm
the total horizontal distance is 3 cm
For the vertical distance, it is 1 cm
so the distance = $\sqrt{3^2 + 1^2}$
= $\sqrt{10}$

Figure 6. Answer G3 for question in indicator 2

In G3's answer, it can be seen that G3's first step was to present their answer by drawing a cube and identifying what was known about the cube, namely that the width of the front side was 1 cm, the width of the right side was 1 cm, and the width of the back side was 1 cm, and the total horizontal distance was 3 cm. For the vertical distance of 1 cm, the distance is found using the Pythagorean theorem, which is √10. During the interview, G3 explained his answer by saying that he imagined the cube's net, and from the cube's net, he found the shortest path, which was found using the Pythagorean theorem, namely $\sqrt{3^2 + 1^2}$. However, G3 was unable to explain how he arrived at 3 and 1 precisely, nor could he create a representation using cube nets when asked. This indicates that G3 has not yet reached the Object stage but is still at the Process stage. Meanwhile, G1 has not only reached the Object stage but has also reached the Scheme stage for the second indicator. G1's answer can be seen in Figure 7.



The cube is opened, and the nets are as shown below

The shortest path distance with the ant is $\sqrt{17}$ cm

Figure 7. Answer G1 for question in indicator 2

G1 is very capable of choosing and applying the appropriate representation in solving the given problem. G1 also provides a clear explanation of how he made the cube net from the given problem. After making the cube net, G1 drew a straight line from A to B' to determine the distance from A to B, with the condition that it must pass through all sides of the cube. A to B' are connected by a straight line that shows the shortest distance from A to B, so to determine the distance, G1 uses the Pythagorean theorem in the form of $\sqrt{(AA')^2 + (A'B')^2}$, which gives a final result of $\sqrt{17}$. G1 is able to explore and use algebraic and geometric representations well, both in writing and orally. Therefore, in the second indicator, G1 has the ability to think at the Scheme level

3. Using representations to model and interpret physical, social, and mathematical phenomena.

In the third indicator, the highest stage achieved by new students is the process stage, which proves that for indicator 3, students' representation skills are still weak. In this indicator, there are students who did not respond, namely G5. Then, the students who reached the action stage were G5 and G3. G3's answer can be seen in Figure 8.

Given : ←	5. Diket : $r = 30$ cm
$r = 30$ cm	$t = 90$ cm
	$h = \frac{1}{2} r = \frac{1}{2} \times 30 = 15$ cm

Figure 8. Answer G3 for question in indicator 3

From Figure 8, it can be seen that G3 is only able to write or copy what is known from the question and is not yet able to use mathematical representation in modelling oil in a tube either verbally or in writing, which proves that G5's ability has only reached the action stage. Meanwhile, the process stage is achieved by G1 and G4. G4's answer can be seen in Figure 9.

5. Diket : $r = 30$ cm
$t = 90$ cm (Panjang tabung saat horizontal)
h dari lantai $= \frac{1}{2} r = \frac{1}{2} \cdot 30 = 15$ cm
dit : V
Jawab : V = luas penampang minyak × ~~t~~ panjang tabung
(segmen lingkaran)
V = segmen lingkaran × 90 cm

Translation :
Given : $r = 30\ cm$
$t = 90\ cm$ (length of the cylinder when horizontal)
h from the floor $= \frac{1}{2} r = \frac{1}{2} \cdot 30 = 15\ cm$
Question : V
Answer : V = area of the base segment × height of the cylinder
V = area of the base segment ×90 cm

Figure 9. Answer G4 for question in indicator 3

In this answer, G4 wrote down all the important points in the question. G4 also explained that to determine the volume of oil, the area of the segment must be multiplied by the length (height) of the tube. Here, we can see that G4 is able to copy and create a strategy to represent the volume of oil. However, G4 is not yet able to create an equation for the area of the segment, so G1's ability has only reached the process stage.

The results of the study indicate that students' mathematical representation abilities are still relatively low on all three NCTM indicators. The highest average score was on the indicators of selecting, applying, and translating representations (1.76), while the lowest score was on the indicator of using representations to model physical, social, and mathematical phenomena (1.35). These findings are consistent with the study by These results are in line with the findings of Nathan et al. (2002), which show a gap between students' abilities to understand and produce representations, especially between symbolic representations and graphs or table (Nathan et al., 2002). This means that low representation is not only a matter of technique but also a matter of difficulty in moving between representations.

Through a more in-depth analysis via interviews with six students, a varied distribution of APOS cognitive stages was revealed. On the first indicator (organising, recording, and communicating mathematical ideas), students with low abilities tended to only be at the Action stage, characterised by copying procedures or answers without conceptual understanding. This is in line with Huinker's (2015) argument about the importance of representational competence the ability to use various representations meaningfully to understand and communicate mathematical ideas(Huinker, 2015).

In the second indicator (selecting, applying, and translating representations), the same pattern emerged. Low-ability students only wrote down basic information without imagining the problem-solving process, thus stopping at the Action stage. Intermediate-level students were able to reach the Process stage by trying to visualise cube nets, although they were not yet able to consistently connect the visualisation results with symbolic representations. High-ability students demonstrated an understanding of the Schema stage, such as G1, who was able to make cube nets, draw representative lines, and formulate solutions by flexibly combining visual and symbolic representations. These findings are parallel to Fonger's (2011) study, which developed an analytical framework for categorising inter-representation relationships as indicators of representational fluency (Fonger, 2011).

The third indicator (using representations to model phenomena) shows the most significant weakness. Even the best students were only able to reach the Process stage. New students could identify the volume of oil in a tube as the product of the area of the segment and the height, but failed to derive the mathematical equation. This is in line with Duval (2017) view that the biggest obstacle in mathematical representation lies in translation between registers, especially when connecting real-world phenomena with formal notation. Research has also shown that representation mismatches are often caused by students' different epistemological frameworks for mathematical and graphical representations, as demonstrated by Maries et al. (2020) in the context of physics (Maries et al., 2016).

Overall, the results of this study indicate that students' mathematical representation abilities are closely related to the APOS cognitive stage. Students who stop at the Action stage tend to only produce procedural representations, while those who reach the Schema stage are able to effectively integrate representations and solve mathematical problems or contextual phenomena. This study also expands the use of APOS theory as an analytical tool for mathematical representation abilities based on the stages in APOS theory. These findings imply that learning needs to be explicitly designed to encourage students to transition from Action to Schema through the use of integrated multi-representations. This is in line with global research that emphasises the importance of representational fluency as the basis for developing deeper conceptual (Borji et al., 2018; Hill & Sharma, 2015; Okoye-Ogbalu & Nnadozie, 2023; Tatira & Mukuka, 2024; Trigueros & Martínez-Planell, 2010).

4. Conclusion

This study shows that students' mathematical representation abilities are still at a relatively low level, especially in terms of using representations to model and interpret physical, social, and mathematical phenomena. Analysis based on APOS theory shows that low-ability students tend to stop at the Action stage, moderate-ability students are at the Process stage, while high-ability students can reach the Object and even Schema stages. These findings confirm that the quality of representation is closely related to the stages of students' cognitive development, where the transition from Action to Schema requires a more flexible and meaningful integration of multiple representations.

The implication of this study is the importance of designing learning strategies that encourage students to explicitly connect various forms of mathematical representations, whether visual, symbolic, verbal, or contextual, so that they can build a more complete conceptual understanding. Further research is recommended to expand the context of the study to other mathematical topics, integrate digital technology-based approaches, and involve more participants from different institutions to obtain a more comprehensive picture of the relationship between representation abilities and cognitive stages according to APOS theory. However, this study is limited by its small sample size and focus on a single topic in geometry, which may restrict the generalisability of the findings. In addition, the qualitative nature of the research means that interpretations are context-dependent and may vary across different learning environments. Future research could employ mixed-method or longitudinal designs to examine how students' representational fluency and cognitive transitions develop over time, as well as explore how technology-enhanced learning environments might support these cognitive progressions

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within the APOS framework.

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